VARIOUS FORMULAE FOR THE GENERATION OF PRIME NUMBERS, USING AN ANALYTICAL "BINARY" TECHNIQUE.

George Byron Koch

A prime number is an integer divisible only by itself and 1; it is non-composite, not the product of any of the integers that precede it. By a direct, though curious, translation of this definition into mathematical terms, various formulae, including the nth prime, a prime given the preceding prime, sums of primes in intervals, and the nth prime pair, may be generated.

The formula for the nth prime number, p_n, where p_1=2, p_2=3, etc. (all brackets, braces and parentheses for grouping only):

$$p_n = \sum_{t=2}^{n^2+1} t \left\{ \sin\left[\pi/2 \left| 2n - 2\sum_{r=2}^{t} \left(1 - \sin\left[\pi/2 \left| \prod_{m=0}^{r-2} \prod_{l=0}^{r-2} (ml-r) \right| !\right] \right) \right| ! \right] \right\} \left\{ 1 - \sin\left[\pi/2 \left| \prod_{j=0}^{t-2} \prod_{k=0}^{t-2} (jk-t) \right| !\right] \right\} \qquad (1)$$

The function

$$F(r) = \sin\left[\pi/2 \left| \prod_{m=0}^{r-2} \prod_{l=0}^{r-2} (ml-r) \right| !\right] \qquad (2)$$

tests r for primality. When r is prime, F(r) = 0. When r is composite, F(r) = 1. The function F(r) is of the more general form $F(x) = \sin[(\pi/2)|x|!]$, giving only binary results for x an integer, i.e. F(x) = 1 for $x \neq 0$, and 0 for x = 0, noting that 0! = 1. The function F(x) may be considered as a member of a class of functions giving only binary solutions, and thereby providing a computer-like manipulation of problems.

The product series $\prod \prod (ml\text{-}r)$ [with the above-stated limits] multiplies every integer from 0 to r-2 by every integer from 0 to r-2. If any of these products equals r [meaning r is composite], the specific term of the series where this occurs equals zero, and thus the entire product series equals zero.

It follows finally that for each t from t=2 to n^2+1, the quantity in the left-hand braces is either 0 or 1. It is 1 if and only if r is in the aforementioned interval, and 0 for all other values of r. The quantity in the right-hand braces is 0 for all t composite, and 1 for all t prime. When $t=p_n$, the quantities in both the right- and left-hand braces equal 1. When $t\neq p_n$, at least one of these quantities will equal zero. Hence, the summation series of t is equal to p_n.

*   *   *

DERIVATIVE FORMULAE:

To find a prime q, given the preceding prime p:

$$q = \sum_{t=p+2}^{2^n p} t \left\{ \sin\left[\pi/2 \left| 2n - 2\sum_{r=p+2}^{t} [1-F(r)] \right| ! \right] \right\} \left\{ 1 - F(t) \right\} \tag{3}$$

by simple substitution in (1), noting here that $p<q<2p$ and n=1. Further, for n=2,3,4,..., the above formula for q yields the 2nd, 3rd, 4th,..., successive primes after p.

To find the sum of primes in a given interval: From (2) we find that r[1-F(r)] equals r for r prime, and 0 for r composite. Therefore:

$$\text{Sum of primes} = \sum_{r=d}^{f} r[1-F(r)] \tag{4}$$

for the interval from d to f. If d or f are primes, they are included in the sum.

To find the nth prime pair, where q denotes the first prime, and q+2 the second prime of the pair:

$$q_n = \sum_{t=2}^{n^2+1} t \left\{ \sin\left[\pi/2 \left| 2n - 2\sum_{r=2}^{t} [1-F(r)][1-F(r+2)] \right| ! \right] \right\} \left\{ [1 - F(t)][1 - F(t+2)] \right\} \quad (5)$$

since the product of the quantities in braces will equal 1 if and only if both t and t+2 are prime. This is not a proof of an infinitude of pairs.

George Byron Koch
1800 N Highland
Los Angeles CA 90028

(reproduced from a 1971 paper)